1^{st} order PDE

① Method of characteristic (Lagrange Method)

$$a(x,y,u)\,u_x + b(x,y,u)\,u_y - c(x,y,u) = 0$$

$$\frac{dx}{a(x,y,u)} = \frac{dy}{b(x,y,u)} = \frac{du}{c(x,y,u)}$$

$$f(\phi,\psi) = 0 \longrightarrow u(x,y) - u = 0$$

Ⓘ Canonical Method [Cancelled] ملغي

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Ⓘ Method of speration of Variable

$$f(x,y,u,u_x,u_y) = 0$$ مهم جدا

فك المعادلة على الشكل

$$u = f(x)\,.\,g(y) \Rightarrow u_x = f'g \longrightarrow u_y = fg'$$

$$u = f(x) + g(y) \Rightarrow u_x = f' \longrightarrow u_y = g'$$

Example 2.7.1

Solve the IVP

$$u_x + 2u_y = 0$$

$$u(0,y) = 4e^{-2y}$$

$$u = f(x).g(y)$$

$$u_x = \frac{df}{dx}.g \quad , \quad u_y = \frac{dg}{dy}.f$$

$$\frac{df}{dx}.g + 2f\frac{dg}{dy} = 0$$

Divide by. f.g

$$\frac{\frac{df}{dx}.\cancel{g}}{f.\cancel{g}} + 2\cancel{f}\,\frac{dg/dy}{\cancel{f}.g} = 0$$

$$\frac{df/dx}{f} + 2\,\frac{dg/dy}{g} = 0$$

$$\frac{df/dx}{f} = -2\,\frac{dg/dy}{g} = \lambda \longrightarrow \text{Constant}$$

$$\frac{f'}{f} = \lambda \longrightarrow \int\frac{df}{f} = \int\lambda\, dx$$

$$\therefore \ln f = \lambda x + \text{Constant}$$

$$\therefore \boxed{f = C_1 e^{\lambda x}}$$

$$-2\frac{dg/dy}{g} = \lambda$$

$$\int \frac{dg}{g} = \int \frac{-\lambda}{2}\, dy$$

$$\therefore \ln g = \frac{-\lambda}{2} y + \text{Constant}$$

$$g(y) = e^{\frac{-\lambda}{2} y} C_2$$

$$\boxed{g(y) = C_2 e^{\frac{-\lambda}{2} y}}$$

$$u(x,y) = f(x) \cdot g(y)$$

$$= C_1 e^{\lambda x} * C_2 e^{\frac{-\lambda}{2} y}$$

$$u(x,y) = C e^{\lambda x - \frac{\lambda}{2} y}$$

$$\boxed{u(x,y) = C e^{\lambda(x - \frac{y}{2})}} \longrightarrow \text{General solution}$$

$$u(0,y) = 4e^{-2y} = C e^{\lambda[0 - \frac{y}{2})}$$

$$u(0,y) = 4e^{-2y} = C e^{\frac{-\lambda}{2} y}$$

$$C = 4 \quad , \quad \lambda = 4$$

$$\therefore \boxed{u(x,y) = 4 e^{4[x - \frac{y}{2}]}}$$

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$$u(x,y) = f(x) + g(y)$$

$$u_x = \frac{df}{dx} \quad , \quad u_y = \frac{dg}{dy}$$

$$\frac{df}{dx} + 2\frac{dg}{dy} = 0$$

$$\frac{df}{dx} = -2\frac{dg}{dy} = \lambda$$

$$f = \lambda x + C_1 \quad , \quad g = \frac{\lambda}{2} y + C_2$$

$$u(x,y) = \lambda x - \frac{\lambda}{2} y + C_1 + C_2$$

$$\boxed{u(x,y) = \lambda x - \frac{\lambda}{2} y + C}$$

To find λ and c

$$u(0,y) = 4e^{-y/2}$$

$$u(0,y) = \frac{-\lambda}{2}y + c$$

$$4e^{-y/2} = \frac{-\lambda}{2}y + c$$

λ and c cannot be got.

$u(x,y) = f(x) + g(y)$ is not soutable for their initial condition - - switch

ex solve $y^2 u_x^2 + x u_y = (xyu)^2$

$$u = f(x) \cdot g(y)$$

$$u_x = f' \cdot g \ , \ u_y = f \cdot g'$$

$$y^2 (f')^2 \cdot g^2 + x \cdot f \cdot g' = (xyfg)^2$$

Divide by $(xyfg)^2$

$$\frac{y^2 (f')^2 g^2}{(xyfg)^2} + \frac{xfg'}{(xyfg)^2} = 1$$

$$\frac{(f')^2}{(xf)^2} + \frac{g'}{xfy^2g^2} = 1$$

$$\frac{f'^2}{x^2 f^2} + \frac{g'}{xfy^2g^2} = 1$$

Multiply by $x.f(x)$

$$\frac{f'^2}{xf} + \frac{g'}{y^2g^2} = x.f$$

$$\underbrace{\frac{f'^2}{xf} - xf}_{(x)} + \underbrace{\frac{g'}{y^2g^2}}_{(y)} = 0$$

$$\frac{f'^2}{xf} - xf = -\frac{g'}{y^2g^2} = \lambda$$

$f(x) \Rightarrow \frac{f'^2}{xf} - xf = \lambda \ , \ g(y) \Rightarrow \frac{-g'}{y^2g^2} = \lambda$

$$f'^2 - x^2 f^2 = \lambda x f$$

$$f'^2 - x^2 f^2 - \lambda x f = 0$$

Put $f' = P$

$$P^2 - x^2 f^2 - \lambda x f = 0$$

$$P^2 - x f[x f + \lambda] = 0$$

$$\left(P - \sqrt{xf(xf+\lambda)}\right)\left(P + \sqrt{xf(xf-\lambda)}\right) = 0$$

$$\Rightarrow \frac{-g'}{y^2 g^2} = \lambda$$

$$\int \frac{-dg}{g^2} = \int y^2 \lambda \, dy$$

$$g^{-1} = \lambda \frac{y^3}{3} + C_2$$

$$g = \frac{C_2}{\lambda \, y^3/3}$$

*Can we use the sum.

$$u(x,y) = f(x) + g(y)$$

$$u_x = f', \quad u_y = g'$$

$$y^2 u_x^2 + x u_y = (xyu)^2$$

$$y^2 f'^2 + x g' = [xy(f+g)]^2$$

$$y^2 f'^2 + x g' = x^2 y^2 (f+g)^2$$

Sum is not useful

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VIP

solve

$$u(x+y)\, u_x + u(x-y)\, u_y = x^2 + y^2$$

$$u(x+y)\, u_x + u(x-y)\, u_y - (x^2+y^2) = 0$$

$$a = u(x+y)$$

$$b = u(x-y)$$

$$-c = -(x^2+y^2)$$

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c}$$

$$\frac{dx}{u(x+y)} = \frac{dy}{u(x-y)} = \frac{du}{x^2+y^2}$$

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$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$$

$$= \frac{xa+yc}{xb+yd} = \frac{xa-yc}{xb-yd}$$

$$\frac{-x\,dx}{-xu(x+y)} = \frac{y\,dy}{uy(x-y)} = \frac{+u\,dy}{+u(x^2+y^2)}$$

$$= \frac{-x\,dx + y\,dy + u\,dy}{\cancel{-x^2u} \cancel{\mp xyu} + \cancel{xyu} - \cancel{uy^2} + \cancel{ux^2} + \cancel{uy^2}}$$

$$= \frac{-x\,dx + y\,dy + u\,dy}{0}$$

$$\int -x\,dx + \int y\,dy + \int u\,dy = 0$$

$$-\frac{x^2}{2} + \frac{y^2}{2} + \frac{u^2}{2} = C_1 = \phi(x,y,u)$$

for $\psi(x,y,u)$

$$\frac{y\,dx}{yu(x+y)} = \frac{x\,dy}{xu(x-y)} = \frac{-u\,du}{-u(x^2+y^2)}$$

$$= \frac{y\,dx + x\,dy - u\,dy}{0}$$

$$\therefore\ y\,dx + x\,dy - u\,du = 0$$

$$\int d(xy) - \int u\,du = 0$$

$$xy - \frac{u^2}{2} = C_2$$

$$\cancel{\psi(x,y)} \quad 2xy - u^2 = \psi(x,y,u)$$

$$f(\phi,\psi) = 0$$

$$-x^2 + y^2 + \cancel{u^2} + 2xy - \cancel{u^2} = 0$$

$$-x^2 + y^2 + 2xy = 0$$